

## Simplified Poisson Pressure Equation (PPE) and Technical Explanation

### The Simplified Equation

For incompressible flow, the pressure field is derived from the Poisson Pressure Equation:

$$\nabla^2 p = -\rho \left( \frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \cdot \frac{\partial v}{\partial y} + 2 \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial x} \right)$$

This simplified form isolates the core pressure-generating mechanisms arising from velocity-gradient interactions. Any spatial change in velocity—shear, rotation, dilation, or deformation—feeds directly into the source term on the right-hand side. The pressure field is then obtained by inverting the Laplacian, distributing these sources in space according to the Green's function of the operator.

### Interpretation in Turbulent Flow

Turbulent eddies contain strong local velocity gradients due to rapid spatial variation in the flow. The PPE captures how these gradients translate into measurable pressure fluctuations:

- **Large eddies:** Their gradients extend over large spatial regions, producing pressure fields with long spatial correlation lengths. The pressure decays slowly with distance, resulting in broad, low-frequency pressure structures.
- **Small eddies:** Their gradients are confined to compact regions, generating pressure signatures that decay rapidly. These pressure fields exhibit short coherence lengths and correspond to higher frequencies in the hydrodynamic spectrum.

### Why This Is Useful

The simplified PPE provides a clear, physically grounded link between eddy size, gradient magnitude, and pressure-field extent. It enables basic analysis of hydrodynamic pressure behavior without full CFD and provides a framework for interpreting coherence, pseudosound reach, and the transition to acoustically propagating pressure disturbances.

### One-Sentence Summary

The simplified PPE demonstrates that pressure originates from velocity-gradient mechanics, and the spatial scale of an eddy governs the spatial reach and coherence of its associated pressure field.